**SAINT IGNATIUS' COLLEGE**

**Trial Higher School Certificate**

# 2004

# MATHEMATICS

**8:50am – 11:55 am**
**Monday 23rd August 2004**

**Directions to Students**

| | |
|---|---|
| • Reading Time : 5 minutes | • Total Marks **120** |
| • Working Time : 3 hours | |
| • Write using blue or black pen. (sketches in pencil). | • Attempt Question 1 – 10 |
| • Board approved calculators may be used | • All questions are of equal value |
| • A table of standard integrals is provided at the back of this paper. | |
| • All necessary working should be shown in every question. | |
| • **Answer each question in the booklets provided and clearly label your name and teacher's name.** | |

**Total marks (120)**
**Attempt Questions 1 – 10**
**All questions are of equal value**

Answer each question in a SEPARATE writing booklet.

---

**QUESTION 1** (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\dfrac{\left(\sqrt{6}+1\right)^2}{\sqrt{6}-1}$ correct to three significant figures. **2**

(b) Simplify $|-8|-|11|$. **1**

(c) Simplify $\dfrac{x}{2}-\dfrac{2x-3}{5}$. **1**

(d) Find the exact value of $\cos\dfrac{\pi}{3}+\cos\dfrac{3\pi}{4}$. **2**

(e) Find a primitive of $x^3-5$. **2**

(f) Express $\dfrac{1}{5+\sqrt{2}}+\dfrac{1}{5-\sqrt{2}}$ in simplest surd form. **2**

(g) In 2003, I travelled 18 480 km in my car, which was 17.5% less than the distance I travelled in 2002. What distance did I travel in 2002? **2**

**QUESTION 2** (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate the following functions:

(i) $\left(7-3x^2\right)^6$. **2**

(ii) $x\tan x$ . **2**

(iii) $\dfrac{x}{\sin 2x}$ . **2**

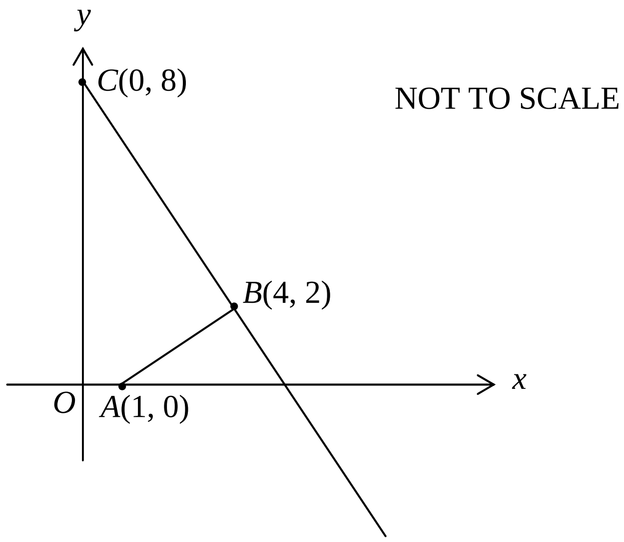
(b) Evaluate the following integrals:

(i) $\displaystyle\int_1^4 \sqrt{x}\,dx$ . **2**

(ii) $\displaystyle\int_0^2 e^{3x}\,dx$ . **2**

(c) Find $\displaystyle\int \frac{6x}{x^2+3}\,dx$ . **2**

**QUESTION 3** (12 Marks) Use a SEPARATE writing booklet. **Marks**




The diagram shows the points $A(1, 0)$, $B(4, 2)$ and $C(0, 8)$ in the Cartesian plane.

(a) Show that the equation of $BC$ is $3x + 2y - 16 = 0$. **2**

(b) Show that $\angle ABC$ is 90°. **2**

(c) Find the length of $AB$. **2**

(d) Find the equation of the circle with centre $A$ that passes through $B$. **2**

(e) The circle in (d) crosses the $y$ axis between the origin and $C$ at point $D$ (not shown on the diagram). Find the coordinates of $D$. **2**

(f) Copy or trace the diagram into your Writing Booklet, and shade the region that satisfies both the inequalities: **2**

$$3x + 2y - 16 \geq 0 \text{ and } y \leq 0 .$$

**QUESTION 4** (12 Marks) Use a SEPARATE writing booklet. **Marks**

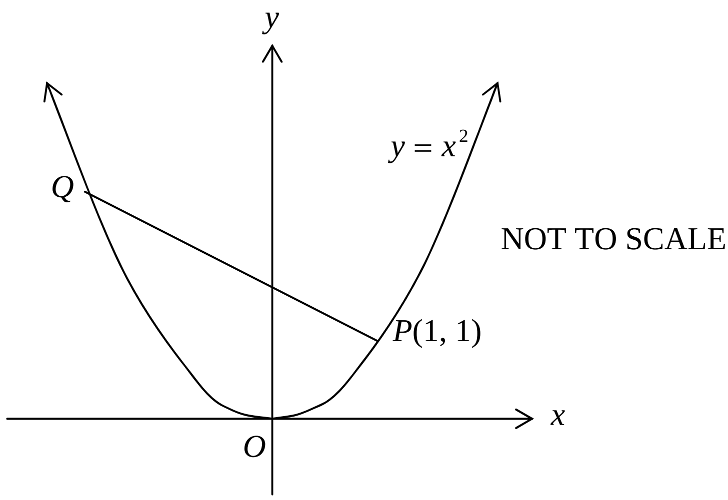
(a) In an arithmetic series, the sixth term is 13 and the tenth term is 1.

(i) Find the first term and common difference. **2**

(ii) Find the sum of the first twenty terms. **2**

(b) A container holds 50 litres of oil. A pump withdraws 10 litres on the first stroke and 7.5 litres on the second stroke. On each future stroke, the pump withdraws ¾ of the amount of the previous stroke. **3**
Show that the container will never be emptied, and find how much oil will finally remain in the container.

(c)




(i) Show that the equation of the normal to the parabola $y = x^2$ at the $P(1,\ 1)$ is $x + 2y - 3 = 0$. **2**

(ii) This normal cuts the parabola again at $Q$. Find the coordinates of $Q$. **3**

**QUESTION 5** (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) The following table shows the values of a function for four values of *x*. **2**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | 1.2 | 3.7 | 5.2 | 1.1 |

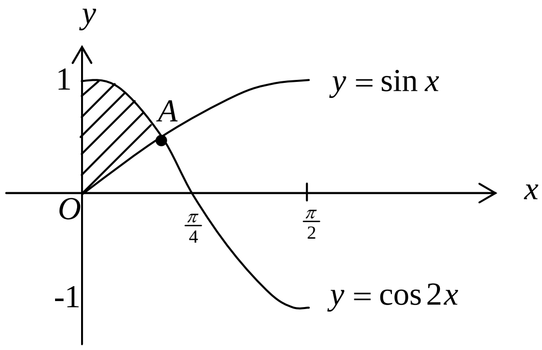
Use the trapezoidal rule to estimate $\int_1^4 f(x)\,dx$.

(b) (i) Copy and complete this table for $f(x) = xe^x$, giving values to 2 decimal places. **1**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | | | |

(ii) Use Simpson's rule to estimate the value of $\int_0^2 xe^x\,dx$. **2**

(c)

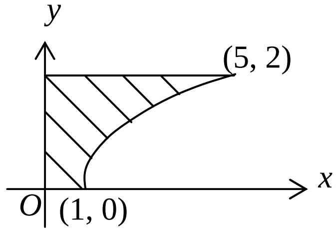



The diagram shows the graphs of $y = \sin x$ and $y = \cos 2x$ for $0 \le x \le \frac{\pi}{2}$. **3**

The graphs intersect at $A\left(\dfrac{\pi}{6}, \dfrac{1}{2}\right)$.

Find the area of the shaded region.

(d)

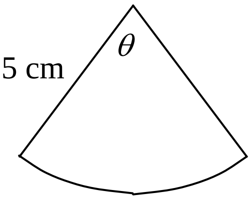



The diagram shows the graph of $y = \sqrt{x-1}$ between (1, 0) and (5, 2). **4**

The shaded region is rotated about the $y$ axis. Find the volume of the solid formed.

**QUESTION 6** (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a)



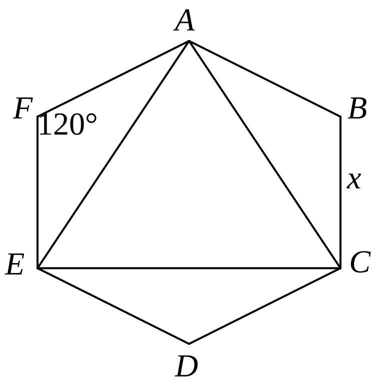

NOT TO SCALE

The radius of a sector of a circle is 11.5cm, and its perimeter is 36.8cm.

(i) Find the size of the angle $\theta$ to the nearest degree. **3**

(ii) Find the area of the sector. **1**

(b)




NOT TO SCALE

*ABCDEF* is a regular hexagon, with each side of length *x*, and each angle 120°. Diagonals *AC*, *AE* and *CE* are drawn.

Copy or trace the diagram into your Writing Booklet.

(i) Explain why $\angle BAC = 30°$. **1**

(ii) Find the size of $\angle EAC$. **1**

(iii) Find the length of *AC*, in terms of *x*, using the Cosine Rule in $\Delta ABC$. **2**

(iv) Find the area of $\Delta ABC$ in terms of *x*. **1**

(v) Find the area of $\Delta ACE$ in terms of *x*. **1**

(vi) Show that the area of $\Delta ACE$ is half the area of the hexagon. **2**

**QUESTION 7** (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Write down the discriminant of $x^2 + kx + (k+3)$. **1**

(ii) For what values of $k$ does the equation $x^2 + kx + (k+3) = 0$ have real and different roots? **2**

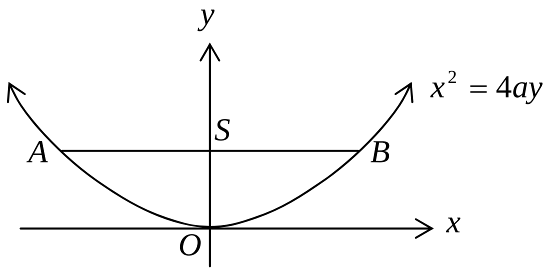
(b) The equation of a parabola is $(x-4)^2 = 12(y+3)$.

(i) Write down the coordinates of the vertex of the parabola. **1**

(ii) What is the focal length of the parabola? **1**

(iii) Write down the equation of the directrix of the parabola. **1**

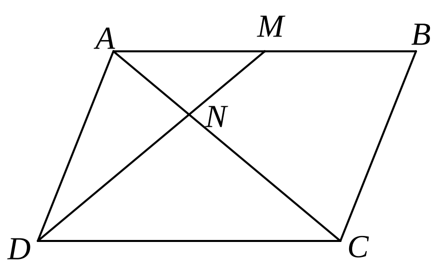
(c)




The diagram shows the graph of the parabola $x^2 = 4ay$, with focus $S$, and $AB$ is the latus rectum (that is, the focal chord perpendicular to the axis of the parabola).

Prove that the length of the latus rectum is $4a$ units. **2**

(d)




*ABCD* is a parallelogram and *M* is the midpoint of *AB*.

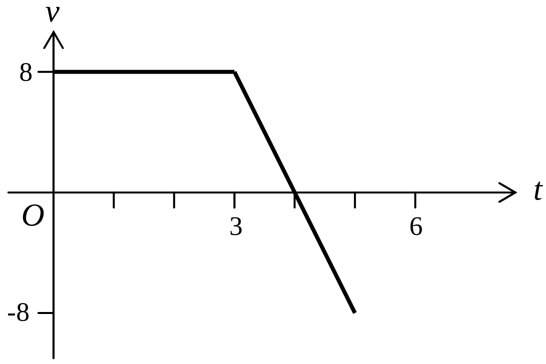
(i) Prove that $\Delta AMN$ is similar to $\Delta CND$. **2**

(ii) Prove that $2AC = 3NC$. **2**

**QUESTION 8** (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve the equation $\cos x + 2\sin x \cos x = 0$, for $0 \le x \le 2\pi$. **3**

(b)




The velocity of a particle ($v$ m/s) at time $t$ seconds is shown in the diagram.

(i) Find the total distance travelled by the particle in the first 5 seconds. **2**

(ii) After how many seconds is the particle the furthest from its starting point? **1**

(iii) Find the acceleration of the particle in the period $3 \le t \le 5$. **1**

(c) The diameter of a tree ($D$ cm) $t$ years after planting is given by the formula

$$D = 60 - 50e^{-0.2t}.$$

(i) Find the diameter of the tree when it is planted. **1**

(ii) Find the diameter after 10 years. **1**

(iii) Find the rate at which the diameter is increasing after 10 years. **2**

(iv) What diameter will the tree eventually approach? **1**

**QUESTION 9** (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Wheat is poured from a silo into a railway truck at a rate $R$ kg/s, given by

$$R = 81t - t^3$$

where $t$ is the time in seconds after wheat begins to flow.

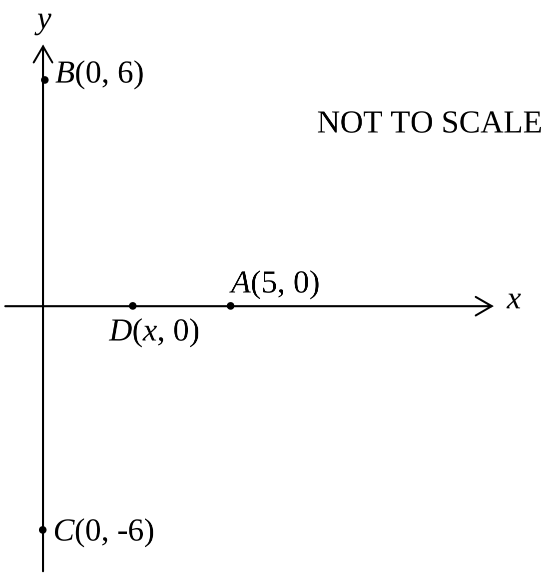
(i) What is the rate of flow when $t = 6$? **1**

(ii) What is the largest value of $t$ for which the expression for $R$ is physically possible? **2**

(iii) Find an expression for the mass $M$ kg of wheat in the truck after $t$ seconds, if initially there was 1 tonne of wheat in the truck. **2**

(iv) Calculate the total weight of wheat in the truck after 6 seconds. **1**

(b)




A company wishes to locate its distribution centre such that its distance fom three different factories is a minimum.
According to a coordinate system, the factories are located at $A(5, 0)$, $B(0, 6)$ and $C(0, -6)$, while the distribution centre lies on the $x$ axis at $D(x, 0)$.

(i) Find an expression, in terms of $x$, for the total distance between the distribution centre and each of the factories, that is: **2**

Distance = $DA + DB + DC$

(ii) Where should $D$ be placed so that this total distance is a minimum. (There is no need to verify that it is a minimum.) **3**

(iii) What is this minimum total distance? **1**

**QUESTION 10** (12 Marks) Use a SEPARATE writing booklet. **Marks**

Consider the function $f(x) = \dfrac{e^x}{x}$.

(a) What is the domain of $f(x)$? **1**

(b) The first derivative of $f(x)$ is $f'(x) = \dfrac{xe^x - e^x}{x^2}$. **2**

Show that the second derivative can be written as:

$$f''(x) = \frac{e^x\left[(x-1)^2 + 1\right]}{x^3}$$

(c) Find the coordinates of the stationary point and determine its nature. **3**

(d) Show that there are no points of inflexion. **1**

(e) For what values of $x$ is the curve concave up and concave down? **2**

(f) Find $\lim\limits_{x \to -\infty} \dfrac{e^x}{x}$. **1**

(g) Sketch the graph of $y = f(x)$. **2**

**End of paper**

## STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

# SAINT IGNATIUS' COLLEGE

## Trial Higher School Certificate

## 2004

## MATHEMATICS

## SUGGESTED SOLUTIONS

## 2004 MATHEMATICS (2U) – QUESTION 1.

(a) $\dfrac{(\sqrt{6}+1)^2}{\sqrt{6}-1} = 8 \cdot 20908 \ldots$

$= 8 \cdot 21$ (3 significant figures) ②

(b) $|-8| - |11| = 8 - 11$

$= -3$ ①

(c) $\dfrac{x}{2} - \dfrac{2x-3}{5} = \dfrac{5x - 2(2x-3)}{10}$

$= \dfrac{x+6}{10}$ ①

(d) $\cos\dfrac{\pi}{3} + \cos\dfrac{3\pi}{4} = \dfrac{1}{2} - \dfrac{1}{\sqrt{2}}$ ②

(e) $\int (x^3 - 5)\,dx = \frac{1}{4}x^4 - 5x$ ②

(f) $\dfrac{1}{5+\sqrt{2}} + \dfrac{1}{5-\sqrt{2}} = \dfrac{5-\sqrt{2}+5+\sqrt{2}}{(5+\sqrt{2})(5-\sqrt{2})}$

$= \dfrac{10}{25-2}$

$= \dfrac{10}{23}$ ②

(g) 82·5 % of distance = 18 480

1% of distance = $\dfrac{18\,480}{82 \cdot 5}$

100% of distance = $\dfrac{18\,480}{82 \cdot 5} \times 100$

= 22 400

I travelled 22 400 km in 2002. ②

**Year 12 Trial HSC Examination**

**Question** 1 **Marker:** TDS

| | Marks Awarded | Marker's Comments |
|---|---|---|
| (a) | 1 | correct calculation |
| | 1 | correct rounding |
| (b) | 1 | correct answer |
| (c) | 1 | correct answer • students lost the mark for not expanding correctly. $-2x - 3 = +6$ |
| (d) | 1 | $\cos\frac{\pi}{3} = \frac{1}{2}$ • didn't have to express as single fraction or rationalise denominator for full marks. |
| | 1 | $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$ |
| e) | 2 | correct answer didn't take mark off for not having '+ C' |
| f) | 1 | correct working |
| | 1 | answer in simplest form |
| g) | 1 | correct working • many students got zero for this question as they found 17·5% of 18480 and added it on. Incorrect method. |
| | 1 | correct answer |

2004 MATHEMATICS (2U) – QUESTION 2.

(a) (i) $\frac{d}{dx}(7-3x^2)^6 = 6(7-3x^2)^5 \times (-6x)$

$= -36x(7-3x^2)^5$ ②

(ii) $\frac{d}{dx}\, x\tan x = \tan x + x\sec^2 x$ ②

(iii) $\frac{d}{dx}\frac{x}{\sin 2x} = \frac{\sin 2x - x.2\cos 2x}{\sin^2 2x}$

$= \frac{\sin 2x - 2x\cos 2x}{\sin^2 2x}$ ②

(b)

(i) $\int_1^4 \sqrt{x}\, dx = \int_1^4 x^{\frac{1}{2}} dx$

$= \left[\frac{2}{3}x^{3/2}\right]_1^4$

$= \frac{2}{3}\left[4^{3/2} - 1^{3/2}\right]$

$= \frac{2}{3}[8-1]$

$= \frac{14}{3}$ ②

(ii) $\int_0^2 e^{3x} dx = \left[\frac{1}{3}e^{3x}\right]_0^2$

$= \frac{1}{3}[e^6 - e^0]$

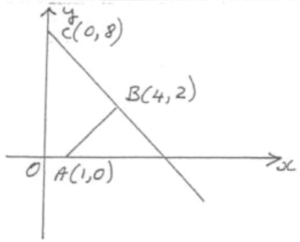
$= \frac{1}{3}(e^6 - 1)$ ②

(c) $\int \frac{6x}{x^2+3} dx = 3\int \frac{2x}{x^2+3} dx$

$= 3\log_e(x^2+3) + C$ ②

| Year 12 Trial HSC Examination Question | Marker: GJA |
|---|---|
| Marks Awarded | Marker's Comments |
| ✓ ✓ | many students couldn't correctly $\frac{d}{dx}(7-3x^2) = -6x$ |
| ✓ ✓ | $x\frac{d}{dx}(\tan x) + \tan x\frac{d}{dx}(x)$ ← product rule answered well. $x\sec^2 x + \tan x$ |
| ✓ ✓ | $\frac{\sin 2x\frac{d}{dx}(x) - x\frac{d}{dx}(\sin 2x)}{\sin^2 2x}$ ← quotient rule; students trouble with $\frac{d}{dx}(\sin 2x) = 2\cos 2x$; correct answer |
| ✓ ✓ | $\frac{2}{3}[x^{3/2}]_1^4$; correct answer. |
| ✓ ✓ | $\frac{1}{3}[e^{3x}]_0^2$ ← this is on the standard integral list! $\frac{1}{3}(e^6-1)$ leave as exact value |
| ✓ ✓ | $3\int\frac{2x}{x^2+3}dx$; $(3)\log_e(x^2+3) + C$ → many students had problems with this coefficient. |

## 2004 MATHEMATICS (2U) – QUESTION 3.




(a) Grad. BC $= \frac{8-2}{0-4} = -\frac{3}{2}$.

Eqn. of BC: $y = -\frac{3}{2}x + 8$

$2y = -3x + 16$

$3x + 2y - 16 = 0$ ②

---

(b) Grad AB $= \frac{2-0}{4-1} = \frac{2}{3}$

Since $\left(-\frac{3}{2}\right) \times \left(\frac{2}{3}\right) = -1$, $AB \perp BC$

i.e. $\angle ABC = 90°$. ②

---

(c) $AB = \sqrt{3^2 + 2^2}$

$= \sqrt{13}$

OR $AB = \left| \frac{3 \times 1 + 2 \times 0 - 16}{\sqrt{3^2 + 2^2}} \right|$

$= \left| \frac{13}{\sqrt{13}} \right|$

$= \sqrt{13}$ ②

---

(d) Equation of circle: $(x-1)^2 + y^2 = 13$ ②

---

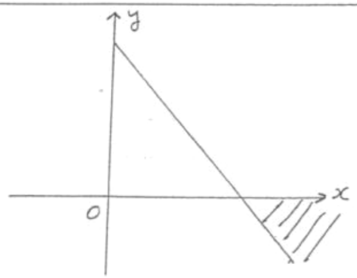
(e) When $x = 0$, $1 + y^2 = 13$

$y = \pm\sqrt{12}$

$\therefore$ D is $(0, \sqrt{12})$ ②

---

(f)




$3x + 2y - 16 \geq 0$

$y \leq 0$ ②

---

**Year 12 Trial HSC Examination**

**Question** 3 **Marker:** FPL.

| Marks Awarded | Marker's Comments |
|---|---|
| a. ✓ ✓ | Generally well done. Students knew the appropriate formula |
| b) ✓ ✓ | Too many students did this using Pythagoras. · Many made the assumption the examiners knew they were referring to part (a) rather than state the gradients explicitly. |
| c ✓ ✓ | Well done. Formula known. |
| d) ✓ ✓ | Well done. Most knew how to get the eqn given centre and radius. |
| e) ✓ ✓ | Many students believe that lines cut the y-axis when y=0. This is false. Many did not state the answer explicitly i.e $D = (0, \sqrt{12})$ |
| f) ✓ ✓ | Some confusion over $y \leq 0$. Consistent with e). |

## Question 4

a) i) $T_6 = 13 \quad T_{10} = 1$

$a + 5d = 13$

$a + 9d = 1$

$4d = -12$

$d = -3$ ✓

$a - 15 = 13$

$a = 28$ ✓

ii) $S_{20} = \frac{20}{2}(2a + 19d)$

$= 10(56 + 19(-3))$ ✓

$= -10$ ✓

b) $S = 10 + 7.5 + \cdots$

$= \frac{10}{1 - 3/4}$ ✓

$= 40.$ ✓

$\therefore$ 10 litres will remain. ✓

e) i) $y = x^2$

$y' = 2x$

at $x = 1$ $M_T = 2$

$M_N = -\frac{1}{2}$ ✓

$y - 1 = -\frac{1}{2}(x - 1)$ ✓

$2y - 2 = -x + 1$

$x + 2y - 3 = 0$

ii) $x + 2y - 3 = 0$

$y = x^2$

$\therefore x + 2(x^2) - 3 = 0$

$2x^2 + x - 3 = 0$ ✓

$(2x+3)(x-1) = 0$

$x = -\frac{3}{2}, 1.$

$x = -\frac{3}{2}$ ✓

$y = \left(-\frac{3}{2}\right)^2$

$= \frac{9}{4}$ ✓

$\left(-3/2, 9/4\right)$

| Year 12 Trial HSC Examination<br>Question FOUR | Marker: NM |
|---|---|
| Marks Awarded | Marker's Comments |
| ✓ ✓ ✓ ✓ | a) i) Well done. Most scored full marks<br>ii) Well done. |
| ✓ ✓ ✓ | b) Many could not determine the correct series.<br>Many did not understand the $S_\infty$.<br>Many could not "show that the container cannot be emptied.<br>(Poorly done). |
| ✓ ✓ ✓ ✓ ✓ | e) i) 1mk for correct gradient<br>1mk for correct $y - y_1 = m(x - x_1)$<br>(Well done by most).<br>ii) 1mk for correct attempt to solve simultaneously.<br>1mk each for $x$ & $y$ coordinates at Q. |

## 2004 MATHEMATICS (2U) - QUESTION 5.

(a) $\int_1^4 f(x)\,dx \doteq \frac{1}{2}[1\cdot2 + 2(3\cdot7 + 5\cdot2) + 1\cdot1]$

$\doteq 10\cdot05$ ②

(b) (i)

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 0 | 2·72 | 14·78 |

$f(x) = xe^x$ ①

(ii) $\int_0^2 xe^x\,dx \doteq \frac{1}{3}[0 + 4\times2\cdot72 + 14\cdot78]$

$\doteq 8\cdot55$ (2 dec. pl.) ②

(c) $A = \int_0^{\frac{\pi}{6}} (\cos 2x - \sin x)\,dx$

$= \left[\frac{1}{2}\sin 2x + \cos x\right]_0^{\frac{\pi}{6}}$

$= \left(\frac{1}{2}\times\frac{\sqrt3}{2} + \frac{\sqrt3}{2}\right) - (0 + 1)$

Area $= \left(\frac{3\sqrt3}{4} - 1\right)$ unit$^2$ ③

(d) $y = \sqrt{x-1}$

$y^2 = x - 1$

$x = y^2 + 1$

$V = \pi\int_0^2 x^2\,dy$

$= \pi\int_0^2 (y^2+1)^2\,dy$

$= \pi\int_0^2 (y^4 + 2y^2 + 1)\,dy$

$= \pi\left[\frac{1}{5}y^5 + \frac{2}{3}y^3 + y\right]_0^2$

$= \pi\left[\frac{32}{5} + \frac{16}{3} + 2\right]$

Volume $= \frac{206\pi}{15}$ unit$^3$ ④

---

**Year 12 Trial HSC Examination**
**Question** 5 **Marker:** CJF

| Marks Awarded | Marker's Comments |
|---|---|
| (a) 1 mark | $\frac{1}{2}$ |
| 1 mark | $1\cdot2 + 2(3\cdot7 + 5\cdot2) + 1\cdot1$ |
| (b) 1 mark | table correctly completed (2 d.p.) |
| 1 mark | $\frac{1}{3}$ |
| 1 mark | $0 + 4(2\cdot72) + 14\cdot78$ |
| (c) 1 mark. | $\int_0^{\pi/6} (\cos 2x - \sin x)\,dx$ or equivalent |
| 1 mark | integration $\left[\frac{1}{2}\sin 2x + \cos x\right]_0^{\pi/6}$ |
| 1 mark. | $\frac{3\sqrt3}{4} - 1$ (or equivalent) |
| (d). 1 mark. | $V = \pi\int_a^b x^2\,dy$ ..... or similar |
| 1 mark | $V = \pi\int_0^2 (y^2+1)^2\,dy$. i.e. $x^2 = (y^2+1)^2$ |
| 1 mark. | $(y^2+1)^2 \to y^4 + 2y^2 + 1$ and integration $\left[\frac{1}{5}y^5 + \frac{2}{3}y^3 + y\right]$ Marks awarded for use of incorrect value of $x^2$ unless trivial. |
| 1 mark | $\frac{206\pi}{15}$. |

## 2004 MATHEMATICS (2U) – QUESTION 6

(a) (i)

$$P = r\theta + 2r$$

$$36.8 = 11.5\theta + 2 \times 11.5$$

$$\theta = 1.2 \text{ radians}$$

$$\theta = 1.2 \times \frac{180}{\pi} \text{ degrees}$$

$$= 69^\circ \text{ (nearest degree)}$$

(3)

(ii)

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \times 11.5^2 \times 1.2$$

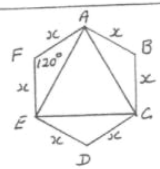
Area of sector $= 79.35 \text{ cm}^2$.

(1)

(b) (i) In $\triangle ABC$, $AB = BC$ $\therefore \angle BAC = \angle BCA$

$\angle ABC = 120^\circ$ $\therefore \angle BAC = 30^\circ$ (angle sum of $\triangle$)

(1)




(ii)

$$\angle EAC = \angle FAB - \angle FAE - \angle BAC$$

$$= 120^\circ - 30^\circ - 30^\circ$$

$$= 60^\circ.$$

(1)

(iii)

$$AC^2 = x^2 + x^2 - 2(x)(x)\cos 120^\circ$$

$$= 2x^2 - 2x^2\left(-\frac{1}{2}\right)$$

$$= 3x^2$$

$$AC = x\sqrt{3}$$

(2)

(iv)

$$\text{Area } \triangle ABC = \frac{1}{2}(x)(x)\sin 120^\circ$$

$$= \frac{\sqrt{3}}{4} x^2$$

(1)

(v)

$$\text{Area } \triangle ACE = \frac{1}{2}(x\sqrt{3})(x\sqrt{3})\sin 60^\circ$$

$$= \frac{1}{2} \times 3x^2 \times \frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{4} x^2$$

(1)

(vi)

$$\triangle AFE + \triangle CDE + \triangle ABC = 3 \times \frac{\sqrt{3}}{4} x^2$$

$$= \text{area } \triangle ACE$$

$\therefore \triangle ACE$ is half the area of the hexagon.

(2)

**Year 12 Trial HSC Examination**

**Question** 6 **Marker:** FPL

| Marks Awarded | Marker's Comments |
|---|---|
| a) i ✓✓✓ | Some confusion over radians and how to find them. Too many did not answer the question that was asked. |
| ii ✓ | Well done. formula known |
| b) i. | well done |
| ii | easily found by most |
| iii | cosine rule was known. That cos 120° has an exact value was not recognised by many. |
| iv. | Area formula known. Exact values not known. |
| v. | as for iv. |
| vi | Reasonably well done. Students are reminded to state their result explicitly and not leave it to the examiner to give the benefit of the doubt. |

## 2004 MATHEMATICS (2U) – QUESTION 7.

(a) (i) $x^2 + kx + (k+3)$

$\Delta = k^2 - 4(k+3)$. ①

(ii) For real & different roots, $\Delta > 0$

$k^2 - 4k - 12 > 0$

$(k-6)(k+2) > 0$

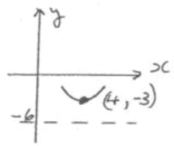
$k < -2,\ k > 6$ ②


(b) $(x-4)^2 = 12(y+3)$.

(i) Vertex $(4, -3)$. ①

(ii) Focal length = 3 units. ①

(iii) Directrix: $y = -6$ ①

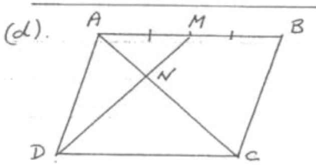



(c) Focus is $(0,a)$. Substitute $y = a$ into $x^2 = 4ay$.

$x^2 = 4a^2 \quad \therefore x = \pm 2a$.

$\therefore$ Length of latus rectum is $4a$ units. ②

(d).




(i) In $\Delta AMN$, $\Delta CND$

$\angle MAN = \angle NCD$ (alt. $\angle$s, $AM \parallel DC$)

$\angle AMN = \angle NDC$ (alt. $\angle$s, $AM \parallel CD$)

$\angle ANM = \angle DNC$ (vert. opp)

} any two of

$\therefore \Delta AMN$ is similar to $\Delta CND$ (2 angles equal) ②

(ii) $AC = AN + NC$

But $\frac{AM}{DC} = \frac{1}{2}$ (data)

$\therefore \frac{AN}{NC} = \frac{1}{2}$ (ratios of lengths in similar triangles)

$\therefore AN = \frac{1}{2} NC$

$\therefore AC = \frac{1}{2} NC + NC$

$2AC = NC + 2NC$

$\therefore 2AC = 3NC$ ②

**Year 12 Trial HSC Examination**
**Question** **Marker:** GJA

| Marks Awarded | Marker's Comments |
|---|---|
| ✓ | $\Delta = K^2 - 4(K+3) = K^2 - 4K - 12$ |
| ✓<br>✓ | $\Delta > 0$ (real and different – learn the different cases)<br>$(K-6)(K+2) > 0$<br>$K < -2$ or $K > 6$ → use a sketch to work out the final answer. |
| ✓<br>✓<br>✓ | $V(4,-3)$<br>$a = 3$<br>$y = -6$<br>→ draw a sketch to work out the answer. |
| ✓<br>✓ | { easiest method let $y = a$<br>Solve $x = \pm 2a$ } → some use focus/directrix definition<br>$2a + 2a = 4a$ |
| ✓<br>✓ | } any two correct reasons (alternate $\angle$s in $\parallel$ lines) (vertically opposite $\angle$s)<br>conclusion + correct reason (equiangular) ← many students lacked this part. |
| ✓<br>} ✓ | Needed to prove $\frac{AN}{NC} = \frac{1}{2}$<br>using properties of $\parallel\parallel\parallel$ $\Delta$s<br>Note $AC = \frac{1}{2}NC + NC$. |

## 2004 MATHEMATICS (2U) - QUESTION 8

(a)
$\cos x + 2\sin x \cos x = 0$
$\cos x(1 + 2\sin x) = 0$
$\cos x = 0$ or $\sin x = -\frac{1}{2}$
$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ ③

(b)(i) Total distance travelled = area between curve & $x$-axis
$= 8\times 3 + \frac{1}{2}\times 1\times 8 + \frac{1}{2}\times 1 \times 8$
Distance = 32 m. ②

(ii) Furthest from starting point after 4 seconds ①

(iii) Gradient $= -\frac{16}{2} = -8$
$\therefore$ acceleration $= -8\ m/s^2$. ①

(c) $D = 60 - 50e^{-0.2t}$

(i) When $t = 0$, $D = 60 - 50\times 1$
Diameter = 10 cm ①

(ii) When $t = 10$, $D = 60 - 50\times e^{-0.2\times 10}$
Diameter = 53·2 cm ①

(iii) $\frac{dD}{dt} = -50\times(-0.2)e^{-0.2t}$
$= 10e^{-0.2t}$
When $t = 10$, $\frac{dD}{dt} = 10\times e^{-2}$
$= 1.35$
Diameter is increasing at 1.35 cm/year. ②

(iv) As $t \Rightarrow \infty$, $e^{-0.2t} \Rightarrow 0$ $\therefore D \Rightarrow 60$
Diameter approaches 60 cm. ①

| Year 12 Trial HSC Examination | |
|---|---|
| Question 8 | Marker: FPL |
| Marks Awarded | Marker's Comments |
| a) ✓✓✓ | Many students divided both sides by cos x. 2 problems with this ① Lose 2 answers (ii) you are dividing by zero at times. |
| b) ✓✓ | Not well done. Distance travelled = \|Area under curve\| not understood. Graph read incorrectly. |
| d) ✓ | acc = gradient well known. |
| c) i ✓ | Well done |
| ii ✓ | Well done |
| iii ✓✓ | Some trouble with the derivative. Some trouble with understanding the question, people wanted to use result from (ii). |
| iv. ✓ | Well done. |

## 2004 MATHEMATICS (2U) – QUESTION 9

(a) $R = 81t - t^3$

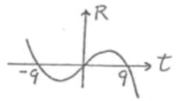
(i) When $t = 6$, $R = 81 \times 6 - 6^3$

Rate $= 270$ kg/s. ①

(ii) Rate must be positive $\therefore 81t - t^3 > 0$

$t(81 - t^2) > 0$

Greatest value of $t$ is 9 seconds.




②

(iii) $M = \frac{81}{2}t^2 - \frac{1}{4}t^4 + c$

When $t=0$, $M = 1000$ $\therefore c = 1000$

$\therefore M = \frac{81}{2}t^2 - \frac{1}{4}t^4 + 1000$ ②

(iv) When $t=6$, $M = \frac{81}{2} \times 6^2 - \frac{1}{4} \times 6^4 + 1000$

Total weight $= 2134$ kg. ①

(b) (i) Distance: $D = DA + DB + DC$

$= (5-x) + \sqrt{x^2+36} + \sqrt{x^2+36}$

$= (5-x) + 2(x^2+36)^{\frac{1}{2}}$ ②

(ii) $\frac{dD}{dx} = -1 + \frac{2x}{\sqrt{x^2+36}}$

For $\frac{dD}{dx} = 0$, $\sqrt{x^2+36} = 2x$

$x^2 + 36 = 4x^2$

$x = \sqrt{12}$ $(x > 0)$.

D should be placed at $(\sqrt{12}, 0)$ ③

(iii) Minimum distance $= (5 - \sqrt{12}) + 2\sqrt{(\sqrt{12})^2 + 36}$

$= 5 - 2\sqrt{3} + 2 \times 4\sqrt{3}$

$= 5 + 6\sqrt{3}$ (or 15.4 1dp) ①

**Year 12 Trial HSC Examination**

**Question** | **Marker:** GJA

| Marks Awarded | Marker's Comments |
|---|---|
| ✓ | easiest mark on the entire exam! $R = 270$ kg/s |
| ✓ ✓ | $81t - t^3 > 0$; $t = 9$ (see sketch). |
| ✓ ✓ | $M = \frac{81}{2}t^2 - \frac{1}{4}t^4 + C$; $M = \frac{81}{2}t^2 - \frac{1}{4}t^4 + 1000$. |
| ✓ | Total Weight = 2134 kg |
| ✓ ✓ | any 1 of the distances correct (DA, DB or DC); $(5-x) + 2(x^2+36)^{\frac{1}{2}}$ |
| ✓ ✓ ✓ | $\frac{dD}{dx} = -1 + \frac{2x}{\sqrt{x^2+36}}$; $4x^2 = x^2 + 36$; $x = \sqrt{12}$. (since $x > 0$). |
| ✓ | let $x = \sqrt{12}$; Min distance $= 5 + 6\sqrt{3}$ [or 15.4 (1 d.p)] |

* look at diagram to work out DA $= 5 - x$

← students had trouble if DA $= (x-5)$ instead of $(5-x)$.

2004 MATHEMATICS (2U) – QUESTION 10.

$f(x) = \frac{e^x}{x}$.

(a) Domain: all $x$ except $x = 0$ (1)

(b) $f'(x) = \frac{xe^x - e^x}{x^2}$

$f''(x) = \frac{x^2[e^x + xe^x - e^x] - [xe^x - e^x]2x}{x^4}$

$= \frac{x^3e^x - 2x^2e^x + 2xe^x}{x^4}$

$= \frac{xe^x(x^2 - 2x + 2)}{x^4}$

$= \frac{e^x[(x-1)^2+1]}{x^3}$ (2)

(c) Stationary point: $f'(x) = 0$ $\therefore$ $e^x(x-1) = 0$

$x = 1$, $y = e$.

$f''(1) = \frac{e^1 \times 1}{1} > 0$

$\therefore$ Stationary point is a minimum at $(1, e)$. (3)

(d) Points of inflexion when $f''(x) = 0$.

$e^x > 0$, $(x-1)^2 + 1 > 0$ for all $x$ $\therefore$ $f''(x) \neq 0$

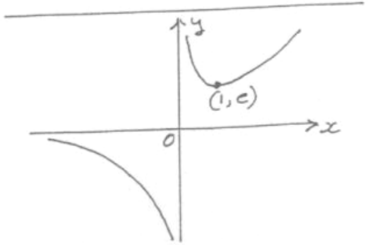
$\therefore$ No points of inflexion. (1)

(e) Concave up: $f''(x) > 0$. Occurs for $x > 0$ (2)

Concave down: $f''(x) < 0$. Occurs for $x < 0$.

(f) $\lim_{x \to -\infty} \frac{e^x}{x} = 0$ (1)

(g)




(2)

**Year 12 Trial HSC Examination**
**Question** 10 **Marker:** TDS

| | Marks Awarded | Marker's Comments |
|---|---|---|
| a) | 1 | correct answer |
| b) | 1 | correct application of quotient rule |
| | 1 | obtaining required result. • once again, careful of expansions where there's a minus sign in front of the bracket. |
| c) | 1 | correct $x$ coordinate |
| | 1 | correct $y$ coordinate |
| | 1 | testing for nature. |
| d) | 1 | had to show why there are no solutions for $f''(x) = 0$. |
| e) | 1 | correct domain for concave up |
| | 1 | correct domain for concave down. • many students tested around the stationary point ($x=1$) for concavity. This led to incorrect solution. • Others assumed that since the curve didn't have a point of inflexion it was always concave up or concave down. • For correct answer, you had to consider what values of $x$ made $f''(x) > 0$ and $f''(x) < 0$. |
| f) | 1 | correct answer |
| | 1 | correct curve in first quadrant, showing turning point. |
| | 1 | correct curve in third quadrant with curve approaching the $x$ axis and $y$ axis. |